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CASCADE MODEL OF GAMMA-RAY BURSTS: POWER-LAW AND ANNIHILATION-LINE COMPONENTS

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ABSTRACT

If, in a neutron-star magnetosphere, an electron is accelerated to an energy of order 10^{11} to 10^{12} eV by an electric field parallel to the magnetic field ("E-parallel"), motion of the electron along the curved field line leads to a cascade of gamma rays and electron-positron pairs. This process is generally believed to occur in radio pulsars, and we have previously proposed that it occurs also in gamma-ray burst sources.

We here present results derived from numerical simulations of the radiation and photon-annihilation-pair-production processes, using a computer code previously developed for the study of radio pulsars. We consider a range of values of initial energy of a primary electron, initial injection position, and magnetic dipole moment of the neutron star. We find that the resulting spectra exhibit complex forms that are typically power law over a substantial range of photon energy, and typically include a dip in the spectrum near the electron gyro-frequency at the injection point. This dip occurs at the junction between the curvature radiation regime and the synchrotron radiation regime. We also estimate the equivalent width to be expected from annihilation radiation produced when positrons impinge on the surface of the star.

We compare the results of a number of models with data for the March 5 1979 gamma-ray burst. We find that we get a good fit to the gamma-ray part of the spectrum, including the equivalent width of the annihilation line, if radiation is produced by primary electrons of energy 5.10^5 MeV injected at a radius of 3.8 stellar radii in the magnetosphere of a neutron star for which the surface magnetic field strength is about 10^{13} G. The requirement that the spectrum should not be modified by Compton

scattering of photons by the cascade particles puts a stringent limit on the distance to the star of about 10 pc.

I. INTRODUCTION

Galactic gamma-ray bursts present many puzzles. One of these concerns the radiation mechanism. The occasional appearance of a redshifted positron-annihilation line indicates that the source of the galactic gamma-ray burst is a neutron star. The most likely mechanism for producing non-thermal gamma-ray radiation from such a source is synchrotron radiation. However, synchrotron radiation is so efficient that, for the particular case of the production of 1 MeV photons in a 10^{12} gauss field, the decay time is only $10^{-16.5}$ s. For some time, this presented the difficulty of understanding how electrons responsible for this radiation, that must have energies at least of order 10^{7} eV, can be accelerated in a time this short or shorter. In order to accelerate an electron to 10^{7} eV in, say, 10^{-17} s, the electron must experience the full effect of an electric field of order 10^{11} esu. However, such a field must be oscillatory (but not necessarily strictly periodic) if it is to increase the transverse energy of electrons, as is required for synchrotron radiation. An example of such a field is one that is circularly polarized and in resonance with the orbital frequency of the electrons, which is about $10^{16.7}$ Hz. Calculation of the electric field strength required to balance radiation damping shows that such a source would be accompanied by soft X-ray radiation in excess of 10^{44} erg s^{-1}. Such a model is untenable, in view of the observational fact that gamma-ray bursts typically emit more energy in gamma rays than in X-rays. Even is this observational fact is ignored, one has replaced the problem of explaining 10^{36} erg s^{-1} in gamma rays by the more difficult problem of explaining 10^{44} erg s^{-1} in soft

X-rays. One can consider other possibilities, but they run into similar problems.

It has been proposed (Sturrock 1986) that this difficulty may be avoided by the hypothesis that observed gamma-ray bursts are the result of the same cascade process that is familiar in the theory of radio pulsars (Sturrock 1971; Ruderman and Sutherland 1975). According to this hypothesis, the primary energization of electrons is ascribed to an electric field (possibly "DC") parallel to the magnetic field. This can energize electrons to sufficiently high energy that they radiate gamma-ray photons by the "curvature" radiation process. If these photons have sufficiently high energy, they can then annihilate in the strong magnetic field of the neutron-star magnetosphere to produce electron-positron pairs. Since these secondary particles have non-zero pitch angles, they can then radiate by the synchrotron process. Photons produced by the synchrotron process may in turn be of sufficiently high energy that they annihilate to produce further electron-positron pairs. This cascade will proceed until the radiated photons are of sufficiently low energy that they may escape from the neutron star magnetosphere.

This model also allows for the production of high-energy emission (above 1 MeV) that is observed in many bursts and is thought to be a common characteristic of gamma-ray burst spectra (Matz et al. 1985). Gamma-ray burst spectra do not exhibit the cut-offs above 1 MeV that would be produced by magnetic (one-photon) pair production if the field strength were of order 10^{12} Gauss or more, but on the other hand some spectra exhibit apparent cyclotron absorption features that indicate field strengths exceeding 10^{12} Gauss. In the cascade model, emission can occur several stellar radii above the neutron-star surface, so that a low field strength in

the emission region is consistent with a high field strength at the surface of the star.

According to this model, most of the positrons that are responsible for the annihilation line are produced with low energies, and most of the continuum gamma rays produced by these electrons and positrons escape from the magnetosphere. These facts leads one to expect a close relationship between the equivalent width of the annihilation line and the strength of the magnetic field where most of the continuum radiation occurs (Sturrock 1986).

We here report the results of a more detailed study of the cascade process, carried out by means of a computer code developed for the study of radio pulsars (Daugherty and Harding 1982). These calculations are discussed in Section II, and their results are discussed in Section III. These studies confirm that the equivalent width of the annihilation line is closely related to the magnetic field strength at the source of the continuum radiation, but the relationship is not exactly that yielded by the simple analysis of Sturrock (1986).

In Section IV, we show that the spectrum of the 1979 March 5 event may be fit to good approximation by a combination of three components: a power-law term, a gaussian term (that one can associate with a red-shifted 511 keV line), and a nearly exponential term such as one would obtain either from bremsstrahlung or from ion synchrotron radiation. In Section V, we compare this fit to models discussed in Section III, and we find a cascade model that provides a good fit to the power-law term and to the equivalent width of the gaussian term. Discussion of the quasi-exponential term is deferred for a later article.

II. CASCADE CALCULATIONS, METHOD

Our calculation of the gamma-ray emission from assumed cascades in neutron star magnetospheres is carried out using a numerical Monte-Carlo procedure described by Daugherty and Harding (1982). The procedure, that is described in detail in that article, was developed for calculation of the gamma-ray emission and the electron-positron flux produced near a pulsar polar cap.

In brief, we assume that an electron is accelerated to an energy $E_{e,i}$ (eV) at radius r (cm) and polar angle θ in the magnetosphere of a neutron star of radius r_*, for which the magnetic field strength at the pole is B_* (gauss). The magnetic field is assumed to be that of a dipole. In the original program, the rotation period P (s) played an important role. In our current calculations, we have set $P = 5$ which is sufficiently long that rotation has a negligible effect on the cascade process.

For simplicity, we assume that there is no further acceleration of the electron after its "injection" with energy $E_{e,i}$. Furthermore, we have considered only injection on or near the dipole equatorial plane ($\theta = 90^o$). Since the primary electron is accelerated by the component of electric field parallel to the magnetic field, it moves along the magnetic field line either away from (positive direction) or toward (negative direction) the neutron star with zero pitch angle. Trajectories in both directions on a closed field line will eventually intersect the surface of the star.

We first calculate the curvature radiation from the primary electron, for an increment of arc length which is sufficiently small that the electron energy changes by only a small amount in that distance, and the radius of curvature changes by only a small amount. The curvature radiation spectrum is divided up into a finite number of frequency bands, and the

"number" (not necessarily an integer) of photons produced in each band is calculated.

The calculation then follows the history of a photon associated with each of these frequency bins. The probability of pair production in the magnetic field is calculated, and pair conversion is assumed to occur when the probability of survival has dropped to 1/e. At the conversion point, both members of the pair are assigned the same direction and half the energy of the parent photon.

The program next calculates the synchrotron radiation from each secondary lepton. We take account of the quantization of this radiation and distribute radiation uniformly with respect to the azimuthal angle of the particle orbit. We make the simplifying assumption that all synchrotron photons are radiated in the orbital plane (thus the pitch angle of the particle remains constant). This is quite accurate at relativistic energies, where most of the photons are produced, but at the lowest particle energies the assumption has the effect of somewhat underestimating the photon energies and of overestimating the final pair energies. In this model, the final energy of an electron or positron is therefore its initial energy parallel to the magnetic field. Since synchrotron radiation is very rapid, calculation of the synchrotron radiation may be made assuming that the magnetic field is locally homogeneous.

The history of photons produced by the secondary leptons is computed in the same way as the history of curvature photons. This leads to further pair production, leading in turn to further synchrotron radiation, etc. We cease to follow the evolution of a branch (or sub-branch or sub-sub-branch, etc.) of the "tree" when either (a) the photon either escapes from the magnetosphere or strikes the surface of the star, or (b) the primary electron

strikes the surface of the star. In this model, since there are no open magnetic field lines, all leptons eventually strike the surface of the star. The energy thus deposited can be a considerable fraction of the initial primary energy and will be either conducted into the star or reradiated in X-rays. This issue will be discussed in a subsequent article.

Although much more information is available from these calculations, for the present purposes we focus our attention on the following output: (a) the spectrum of emergent photons that were produced by curvature radiation; (b) the spectrum of escaping photons that were produced by the synchrotron radiation; (c) the total energy deposited on the surface of the star; and (d) the spectrum and total number of leptons produced by the cascade that originated with a single primary electron.

III. CASCADE CALCULATIONS, RESULTS

A typical spectrum is shown in Fig. 1 for the case GR9. We note that, below about 100 eV, the radiation is due primarily to curvature radiation which has, for this case, a fairly hard spectrum with index -0.8. Above this energy, the radiation is dominated by synchrotron radiation. This component peaks at approximately 200 eV and the intensity then falls off with a power law of index approximately -1.5, as one would expect from the simple theory of electron-positron cascades. This spectrum extends to an energy of approximately 170 MeV, beyond which the photon spectrum (dominated once more by curvature radiation) drops rapidly due to pair production.

Table 1 lists input parameters and output observables for the cascade models that we have investigated. The input parameters include the initial energy of the primary electron (E_{ei}), its initial radius (listed as r/r_*), and

the strength of the magnetic field at the stellar surface (B_*) and at the injection point of the primary electron (B_i). The output observables include the equivalent width ΔE (to be defined below) of the annihilation line; two energies at the lower end of the synchrotron spectrum, E_{L1} where the intensity has a minimum and E_{L2} where it has a maximum; the energy E_U which is the upper energy cut-off of the synchrotron component of the spectrum; and the power law index (n) of the gamma-ray spectrum in two specified energy intervals. The location of E_K is determined by the intersection of the two power laws that approximate the lower range of the synchrotron component (where the spectrum is determined by the cascade process), and the upper range where the spectrum is determined by photon annihilation.

In order for a cascade to occur, the electron energy and the magnetic field strength at injection must be sufficiently high. The condition that the critical curvature photon energy be greater than $2mc^2$ is $E_{e,i} > 10^5\ (r/r_*)^{1/3}$ MeV (the r-dependence comes from the dependence of the curvature-radiation critical energy on the radius of curvature at the injection point), so that the initial primary energy must be at least 10^5 MeV for a cascade to occur. A second condition is that the magnetic field at the injection point must be strong enough that curvature photons have a non-negligible pair-production rate. These conditions may be met with a variety of input parameters: low-energy primaries may be injected near the star in high local magnetic fields to initiate a cascade, but higher-energy primaries injected further from the star can also cause cascades as long as they have sufficient energy ($> \sim 10^5$ MeV) left by the time they reach the higher-field-strength regions.

The positrons that strike the surface of the star will annihilate to produce, for each positron, a pair of photons that we take to be nominally at 511 keV. Hence, for each cascade, the number of photons emitted in the 511 keV line is equal to the number of leptons in the cascade. (N_L). We may therefore characterize the emission by an "equivalent width" defined by

$$\Delta E = \frac{N_L}{dN/dE} \tag{3.1}$$

where dN/dE is the value of the photon number spectrum at 511 keV.

By studying the range of models listed in Table 1, we have studied the dependence of the equivalent width on the parameters of the cascade. From simple cascade theory, we would expect that, approximately,

$$\log \Delta E = \frac{1}{2} \log B_i - 1.3 . \tag{3.2}$$

However, equation (3.2) assumes that all pairs are capable of radiating synchrotron photons at 511 keV. This is not the case for particles having kinetic energies below mc^2 (Lorentz factors below ~2) or Lorentz factors such that the critical energy, above which the synchrotron spectrum falls off exponentially, is below mc^2 , that corresponds to the condition $10^{-13.6} \gamma^2 B < 1$. Furthermore, the primary electron travels from the injection point to the surface of the star, so that it encounters magnetic field strengths much in excess of the injection value B_i. For all these reasons, the dependence of equivalent width on the input parameters may depart significantly from that expected on the basis of the simple theory.

We find that the cases listed in Table 1 may be fit by the relation

$$\log \Delta E = \frac{1}{2} \log B_i - 0.53 \qquad (\sigma = 0.14) \tag{3.3}$$

with the rms error indicated. We find empirically that a somewhat better fit is provided by the relation

$$\log \Delta E = 0.6 \log B_* - 1.1 \log (r/r_*) - 1.99 \quad (\sigma = 0.07), \tag{3.4}$$

where B_* is the value of the magnetic field strength at the surface of the star. The equivalent width appears to be insensitive to the energy of the primary electron, provided that the energy is high enough to give rise to a substantial cascade, i.e. higher than 10^{11} eV.

E_{L1} and E_{L2} are determined mainly by the gyrofrequency at the injection point. We find that

$$\log E_{L1} \approx \log B_i - 8.31 \qquad (\sigma = 0.12) \tag{3.5}$$

$$\log E_{L2} \approx \log B_i - 7.92 \qquad (\sigma = 0.18)\,. \tag{3.6}$$

The location of the upper energy cut-off is determined by the pair-production escape condition, and we find that

$$\log E_U \approx -\log B_i + 18.68 \qquad (\sigma = 0.18) \tag{3.7}$$

As one would expect from the above equations, the observable quantities E_{L1} and ΔE should be related. We find, from the range of models stated, that, approximately,

$$\log \Delta E = 0.5 \log E_{L1} + 3.62 \qquad (\sigma = 0.13)\,. \tag{3.8}$$

Although the power-law index is - 1.5 for the simple model of a cascade, for various reasons the spectrum departs from the simple model. In cascades with very little pair production, primary curvature radiation dominates the gamma-ray spectrum, resulting in a spectral index lower than 1.5. In cascades with copious production of pairs, the gamma-ray spectrum is dominated by synchrotron radiation, the spectral index of

which will be larger than 1.5 for pair spectral indices greater than unity. In order to compare models with the 1979 March 5 event, it is convenient to determine the best power law fit over the range 30 keV to 1 MeV. These values are shown in Table 1, and we see that they range from approximately 1.25 to approximately 2.0.

We find that there is a good correlation between the value of this index and the value of B_i, the strength of magnetic field at the injection point. These quantities are related approximately by

$$n(.03 - 1\,\mathrm{MeV}) \approx 0.22 \log B_i - 0.81 \qquad (\sigma = 0.11)\,. \tag{3.9}$$

Also listed in Table 1 are the spectral indices for power-law fits over the interval 1 - 10 MeV, which range from 1.3 to almost 3. In cases where few pairs are produced, the spectrum flattens at the high energies due to the dominance of curvature radiation in the 1 - 10 MeV region. The SMM-GRS instrument has recorded and analyzed a large number of burst spectra at energies above 1 MeV (Matz 1986). Power laws with indices between 1.5 and 3 appear to give the best fits in about 80% of the events.

IV. THE 1979 MARCH 5 EVENT

This event is of special interest in being one of the most intense and in displaying detailed time variations and spectral information (although the event could of course be in a class of its own).

The spectrum, as determined from the data of Mazets et al. (1982), is shown in Fig. 3. It appears to comprise three distinct components: (a) a low-energy component that is exponential or almost exponential; (b) an approximately power law component; and (c) a "line" component near 430 keV. The last component is conventionally interpreted as the redshifted electron-positron annihilation line at 511 keV. This shift corresponds to

$$z = \frac{GM_*}{R_* c^2} = 0.19 \tag{4.1}$$

This is a reasonable redshift for radiation escaping from the surface of a neutron star. It would correspond, for instance, to a radius $R_* = 10^6$ and a mass $M_* = 10^{33.42}$, i.e. a mass of approximately $1.3M_O$. This well known fact is one of the most direct pieces of evidence that gamma-ray bursts originate at neutron stars.

We have made a least-squares fit of the above spectrum to two models. The first model is based on the assumption that the low energy component is due to optically thin bremsstrahlung radiation. We can fit the fluence F ($keV^{-1}\ cm^{-2}$) to the form

$$F_E = U(E/V)^{-1} e^{-E/V} + WE^{-n} + Xe^{-(E-Y)^2/Z^2} \tag{4.2}$$

and we find that the best least-squares fit corresponds to the values

$$U = 749 \quad V = 39.4 \quad W = 30{,}000 \quad n = 1.73$$
$$X = 1.17 \quad Y = 430 \quad Z = 70$$

where, for this analysis, photon energy E is measured in keV. Figure 3 shows this least-squares fit in relation to the actual data.

We have also examined the possibility that the low-energy part of the spectrum may be due to ion synchrotron radiation. We have therefore examined the alternative form

$$F_E = U(E/V)^{-1/2} e^{-E/V} + W\,E^{-n} + X\,e^{-(E-Y)^2/Z^2}. \tag{4.3}$$

For this functional form, we find that the best least-squares fit yields the values

$$U = 1{,}400 \quad V = 28 \quad W = 30{,}000 \quad n = 1.73$$
$$X = 1.17 \quad Y = 430 \quad Z = 70\,.$$

The form (4.3) gives us a slightly better fit to the data than the form (4.2), although the difference is probably not statistically significant.

From either form, we may compute the total fluence due to the third component (the electron-positron annihilation component). This is found to be

$$F_L = \int dE \ X e^{-(E-Y)^2/Z^2} \approx 145 . \tag{4.4}$$

On the other hand, at 430 keV, the power law contribution to F_E is given by $F_{E,P} = 0.83$. Hence the equivalent width, for this event, is $\Delta E \approx 174$ keV.

V. MODEL OF POWER-LAW AND ANNIHILATION-LINE COMPONENTS

As we saw in Section IV, the spectrum of the March 5 event has three main components: the low-energy component that is almost exponential, the power-law component, and the component that is interpreted as being produced by positron annihilation. In this article, we consider only the second and third components. The low-energy component will be discussed in Part II that will be published at a later date.

In Section III, we presented the results for a range of models specified by the value of the magnetic-field strength at the surface of the star, and the radius at which acceleration occurs. Table 1 lists, among other data, the index of the power-law fit over the interval 30 keV to 1 MeV, and the equivalent width of the 511 keV positron-annihilation line.

We found, in Section IV, that the spectrum of the March 5 burst may be described, to good approximation, either in the form (4.2) or (4.3). In either case, the power-law section of the spectrum has an index of 1.73, and the equivalent width of the emission line is about 174 keV. On comparing

these values with corresponding values for the models discussed in Section III, we find that both quantities can be matched, to good approximation, by the model GR19, ($B_* = 10^{13}$, $r/r_* = 3.78$, $E_{ei} = 5.10^5$ MeV), that yields an index of 1.77 and an equivalent width of 170 keV. The next best fit is given by GR1 ($E_{ei} = 5.10^5$ MeV, $r/r_* = 3.0$, $B_* = 5.10^{12}$), for which the index is 1.77 and the equivalent width is 144 keV.

We need to estimate the number of primary electrons N_e required to match the observed fluence of the March 5 event. To this end, we represent the photon spectrum produced by a single primary electron as $N_1(E)dE$ where, to be compatible with the analysis of Section IV, E is measured in keV. Then the fluence that would be observed at a distance D_{pc} (parsec) from the source is given by

$$F(E) = \frac{N_e N_1(E)}{4\pi\,(10^{18.49}\,D_{pc})^2} \quad . \tag{5.1}$$

If, for comparison with equations (4.2) and (4.3), we represent the power-law part of $N_1(E)$ as

$$N_1(E) = W_1 E^{-n}, \tag{5.2}$$

we then find that the number of electrons required to match the observational data is given by

$$N_e = 10^{38.08}\, D_{pc}^{\;2}\, W\, W_1^{-1}\,. \tag{5.3}$$

For the case that gives the best fit to the March 5 spectrum, we find that $W_1 = 10^{6.4}$, so that equation (5.3) becomes

$$N_e = 10^{36.2}\, D_{pc}^{\;2}\,. \tag{5.4}$$

Our calculation of the photon spectrum has been based on the assumption that (apart from being absorbed by the star itself) photons may

propagate freely from the region in which they are produced. For this to be the case, the optical depth for electron scattering, τ_e, must be less than unity.

The scattering optical depth of a photon may be expressed as

$$\tau_e = \int_0^{s_{max}} \sigma_c n_e \left(1 - \beta \cos \theta(s)\right) \, ds , \tag{5.5}$$

where σ_c is the Compton scattering cross section, n_e is the number density of electrons and positrons, β is the speed of the particles (normalized to the speed of light), s is the distance along the photon path, and $\theta(s)$ is the angle between the velocity vectors of the photon and of the particles through which it moves. If the electrons and positrons are moving along magnetic field lines with negligible pitch angles, and if the photon is initially emitted at the equatorial point of a field line, in a direction parallel to the field line, then the (small) angle θ is given approximately by

$$\theta(s) \approx \frac{s}{R_c} \approx \frac{3s}{r} , \tag{5.6}$$

where r is the radial coordinate and R_c is the radius of curvature near the equator.

If a is the width of the electron-positron beam, then the upper limit of the integral in equation (5.5) is given by

$$s_{max} \approx (ar/3)^{1/2} . \tag{5.7}$$

Since the cascade particles have relativistic energies, we use the Klein-Nishina cross section in the relativistic limit, given by

$$\sigma_c = \frac{3}{8}\,\sigma_T\, x^{-1} \ln(2x) \quad (x >> 1)\,, \tag{5.8}$$

where

$$x = 2\,\gamma_e\,\gamma_\phi \cos\theta\,. \tag{5.9}$$

In these equations, σ_T is the Thompson cross section, $\gamma_e m_e c^2$ is the electron or positron energy, and $\gamma_\phi m_e c^2$ is the photon energy.

Scattering by primary electrons will not be important since their high energy greatly decreases the scattering cross section. The pair energies are in fact distributed in a spectrum with number index approximately -2, but we simplify our estimate by taking an average energy of $\gamma = 2$ for the electrons and positrons.

The electron-positron number density can be estimated from our earlier estimate of the number of primaries N_e needed to give the observed flux:

$$n_e \approx \frac{N_e\,M_p}{\pi\, a^2\, c\Delta t}\,, \tag{5.10}$$

where M_p is the cascade multiplicity factor (the number of secondary particles produced per primary electron) and Δt is the burst duration. By assuming that $s << r$, so that θ is small, we find that the scattering optical depth is given approximately by

$$\tau_e \approx \frac{9}{32.3^{3/2}.\pi}\;\frac{\sigma_T}{\gamma_e\gamma_\phi}\;\ln(4\gamma_e\gamma_\phi)\;\frac{M_p N_e}{c\Delta t\;\; r^{1/2} a^{1/2}}\,. \tag{5.11}$$

This may be expressed numerically as

$$\tau_e \approx 10^{-36.4}\,\gamma_e^{-1}\,\gamma_\phi^{-1}\,\ln(4\gamma_e\,\gamma_\phi)\quad\frac{M_pN_e}{\Delta t\,r^{1/2}\,a^{1/2}}. \tag{5.12}$$

Our preferred model (GR19) for the March 5 burst has $M_p = 10^{3.8}$. On using equation (5.4), considering the case that $\gamma_e = 2, \gamma_\phi = 2$, and requiring that τ_e be less than unity, we arrive at the requirement

$$D_{pc} < 10^{-2.2}\,r^{1/4}\,a^{1/4}. \tag{5.13}$$

For the case GR19, injection occurs at $r = 10^{6.6}$ so that

$$D_{pc} < 10^{-0.5}\,a^{1/4}. \tag{5.14}$$

Hence if $a = 10^6$, $D_{pc} < 10$; if $a = 10^4$, $D_{pc} < 3$.

We see that the large fluence and short time scale of the March 5 impulsive phase implies that it must be quite close by, if the cascade model is applicable. For more typical bursts, the fluence is smaller by a factor of 1000, and the time scale longer by a factor of 10. Hence the distances to more typical bursts will be larger than those estimated by a factor of order 100.

VI. DISCUSSION

We have seen that, in general, the cascade model leads to radiation spectra that resemble the spectra of gamma-ray bursts in several ways. The spectra often exhibit a power-law behavior over a substantial range of energy, and emission can extend to tens or hundreds of MeV. The spectra often display a minimum roughly in the range keV to tens of keV. This minimum is due to the transition from curvature radiation to synchrotron radiation, and occurs at approximately the electron gyro-frequency at the

injection point. It is possible that this minimum may in some cases be the correct interpretation of the "absorption features" found in the spectra of some gamma-ray bursts.

For the cases studied, we obtain power-law indices in the range 1.3 to 2.05. Some gamma-ray bursts display spectra that have smaller power-law indices, that is, spectra that are flatter or "harder". A more realistic version of the present model that incorporates injection of energetic electrons over a range of radii (rather than at a single radius) will lead to a wider variety of spectra, including (we believe) spectra that have power-law indices substantially smaller than the value 1.5 (the "canonical" value for cascades).

We have found that the power-law and annihilation-line components of the spectrum of the March 5 event may be matched quite satisfactorily by the cascade model with reasonable values of the magnetic field and radius of injection of high-energy electrons.

When the cascade model was first proposed (Sturrock 1986), it was suggested that the parallel electric field required to accelerate the primary electrons might be developed as a result of field-line reconnection, such as is believed to occur in solar flares. Further study of this possibility would require the extension of reconnection theory to magnetic fields of very high strength, with plasmas of low density and possibly relativistic energies.

However, there is an alternative process that might lead to the required electric-field configuration. Colgate and Petschek (1981) have considered the fate of a comet or asteroid that makes a direct impact with a neutron star. They attribute the gamma-ray burst to the expulsion of hot gas from the surface of the star when the impact occurs. The combined effects of the gravitational and magnetic fields of the neutron star have the

effect of distorting the object into a thin sheet when it is in the neighborhood of the star.

If the magnetic field penetrates the object to a depth b, a potential difference given by

$$\Delta \phi \approx b \ \frac{v}{c} B \tag{6.1}$$

will develop across the thin sheet of magnetic-field lines that diffuse into the object. Colgate and Petschek estimate that b is of order 10^{-2} cm. For the values $B = 10^{12}$ and $v = 10^{10}$, we find that a potential drop of 10^{12} V develops across this magnetic sheet. This induction effect at the object will result in a corresponding potential drop along the magnetic-field lines between the object and the surface of the star, as in the model proposed by Goldreich and Lynden-Bell (1969) in their discussion of the interaction of Io with Jupiter's magnetic field.

In a forthcoming article, we shall discuss this concept in more detail, and also discuss possible interpretations of the low-energy part of the spectrum and how these interpretations may be related to the cometary impact hypothesis.

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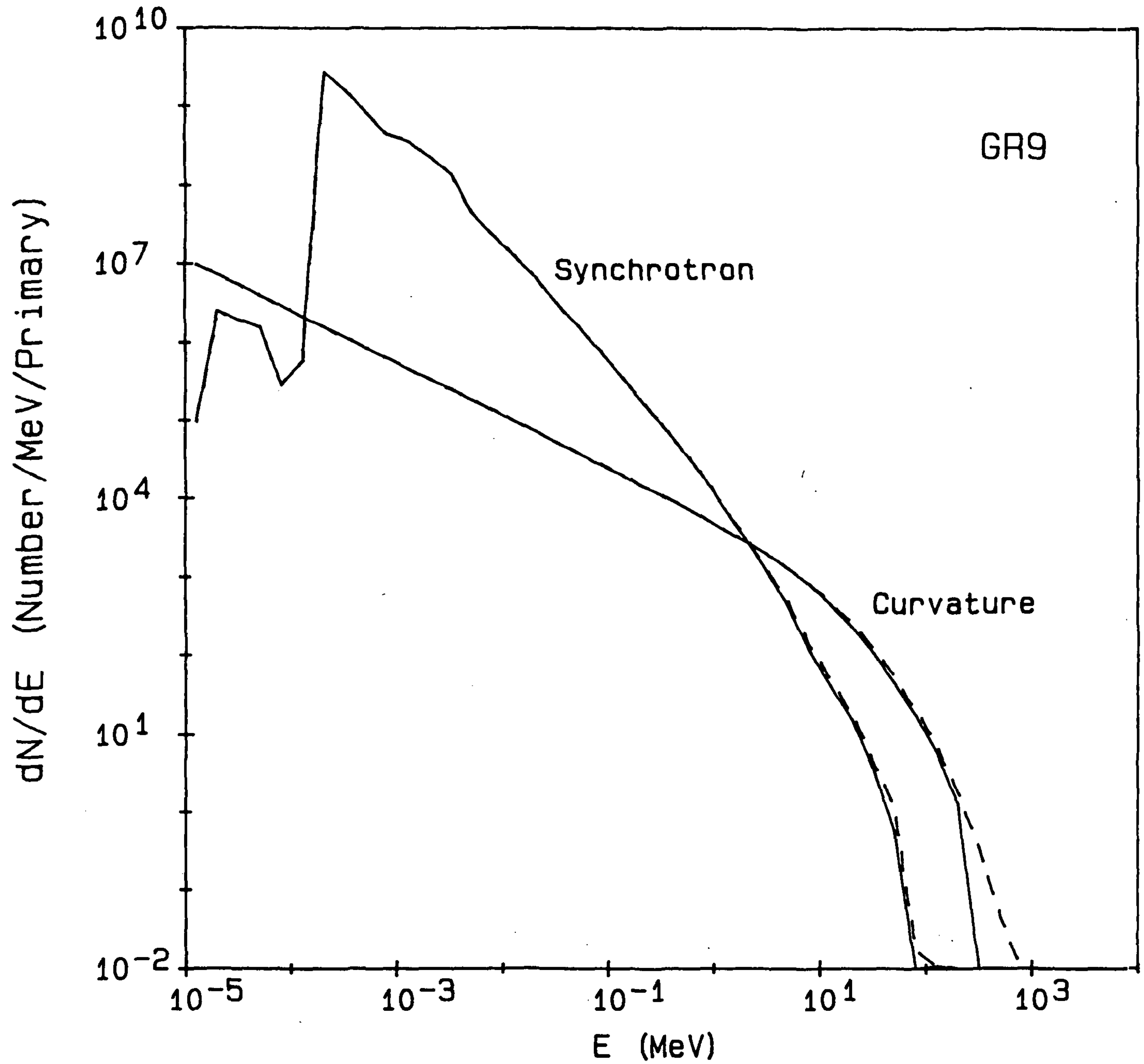
CAPTIONS

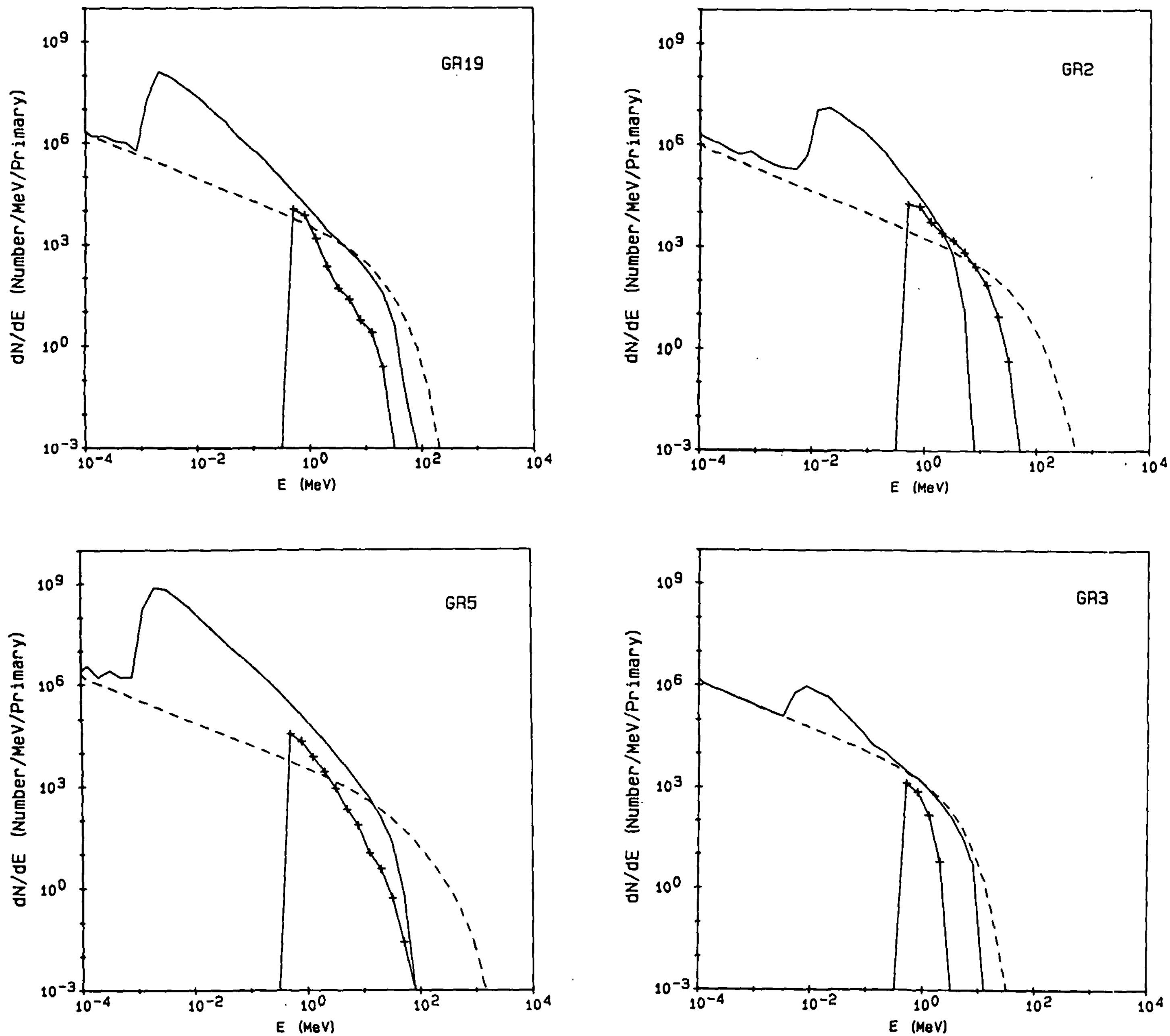
Figure 1. Cascade spectra for the case GR9, showing separate contributions from curvature radiation by the primary electrons and from synchrotron radiation by the secondary electrons and positrons. The broken line gives the spectrum of radiation emitted by the particles, and the solid line gives the spectrum of the radiation that escapes from the star and its magnetosphere.

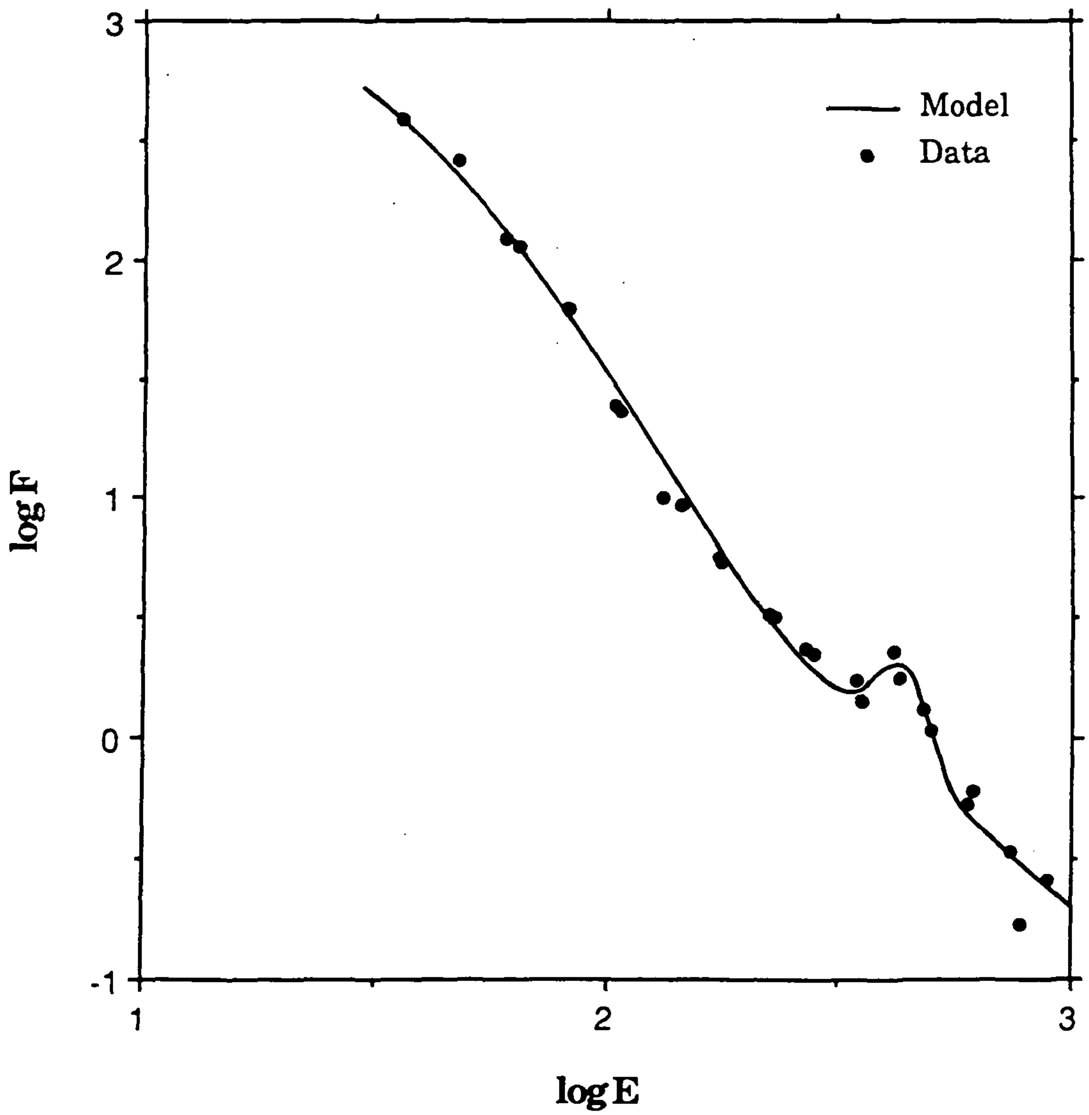
Figure 2. Cascade spectra for four cases identified in Table 1. The broken line is the curvature radiation, and the solid line is the total (curvature plus synchrotron) radiation that actually escapes from the star. The crosses indicate the electron-positron spectrum.

Figure 3. Points indicate the spectrum of the burst-phase fluence of the 1979 March 5 event based on the data of Mazets et al. (1982). The solid line gives the best least-squares fit of the form given by equation (4.2).



GR9
Synchrotron
Curvature
dN/dE (Number/MeV/Primary)
E (MeV)
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-3}
10^{-1}
10^{1}
10^{3}


GR19
GR2
GR5
GR3
dN/dE (Number/MeV/Primary)
E (MeV)
10^{9}
10^{6}
10^{3}
10^{0}
10^{-3}
10^{-4}
10^{-2}
10^{2}
10^{4}



Model
Data
3
2
1
0
-1
log F
1
2
3
log E

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